Retro-directive Noise Correlation Radar with Extremely Low Acquisition Time

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Abstract — **A new retro-directive antenna based search radar system has been introduced. The suggested system uses a novel noise correlation technique. Simulation and analytical results show *an order of magnitude improvement in the acquisition time* of the radar when compared to a phased array antenna based radar system with same specifications, except transmit power. To the best of knowledge of the authors, no radar of a comparable acquisition time has been designed to date. The power versus acquisition time tradeoff has been compared with the phased array radar for evaluating the performance of the system. The radar system is self-tracking due to retro-directivity of the antenna array, and is much easier to implement, as it does not require any phase shifters etc.**

I. Introduction

In the recent years, there has been tremendous increase in research being carried out on beam forming arrayed antennas, particularly for communication and radar system applications. Retrodirective antennas, also known as self-phasing antennas, appear to be attractive solutions for some of these applications. A retrodirective antenna is an antenna that transmits in the direction of the incoming electromagnetic signal without a priori knowledge of the direction of the incoming signal.

To achieve retrodirectivity, the incoming signal is phase-conjugated and retransmitted (with or without modulation), so that the transmitted signal goes back in the same direction as the incoming signal. In the Van Atta array [1], phase-conjugation is achieved by connecting conjugate pairs of antenna elements by equal lengths of transmission lines. A more elegant technique is by using heterodyne mixing of the incoming signal with a carrier of twice its frequency [2]. Retrodirective antennas using this technique have recently been demonstrated in [3]. Full duplex digital communications system using retrodirective antennas has been demonstrated in [4]. Frequency stability for tracking in a mobile environment using retrodirective signaling has also been studied in [5].

In this paper, we propose a new retrodirective antenna based radar, which takes advantage of the retrodirective property to accelerate the detection of targets. Initially the antenna transmits an omni-directional pulse. As the antenna receives the reflected pulse from the target, the next transmitted pulse has some directivity towards the target (though small, as the SNR is very low initially) because of the retrodirectivity of the antenna. In successive pulses, better directivity improves the received SNR, which in turn improves the directivity. As a result, there is a significant improvement of the SNR to a certain limited value.

This paper also introduces a new noise correlation technique. Since each array element receives very low SNR, transmitting the same signal after amplification has a lot of noise in it. So if the first pulse transmitted was a sinusoid, the successive pulses have a lot of noise in them, which is added by the receiver. The noise part of the transmitted power goes as a waste. We can do much better by using noise itself as our signal. In this case, all the elements transmit the band limited random noise present at the input of the receiver for the first pulse. Since the initial noise at each element is uncorrelated, the power transmitted in the first pulse is omni-directional. As discussed earlier, the directivity towards the target builds up after a few pulses. Now the signal received from the target, though appearing as random noise, is correlated at each element receiver output. By cross correlating this signal between different elements, we can get the probability of detection of the target.

This "noise correlation" technique is novel in the sense that it doesn't require any memory and/or variable delay line for storing the transmitted noise signal, as in [6]. The direction of the target can be estimated from the relative phase difference of the signal between consecutive array elements.

II. Description of The Radar System

The proposed radar system consists of a retrodirective antenna with separate transmit and receive array elements. The transmit and receive elements have to be physically separate because the transmitted and received signals are continuous wave and are at the same frequency, thus demanding high isolation, which is difficult to achieve otherwise. Let us consider an antenna array of size $2n^2$, i.e. nxn transmit and nxn receive elements, as shown in Fig 1. The spacing between the antenna elements is kept $\lambda/2$ so that there are no grating lobes and mutual-coupling effects are tolerable. The

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transmit elements *(i, j)* correspond to the receive elements *(i, j)* if the antenna uses heterodyne mixing for phase conjugation. For a Van Atta array, the transmit elements *(i, j)* correspond to the receive elements *(n-i+1, n-j+1)*. In this case, it is important to keep the lengths of the transmission lines connecting the elements the same for all the element pairs, so that the relative phase differences introduced by the transmission lines are zero.

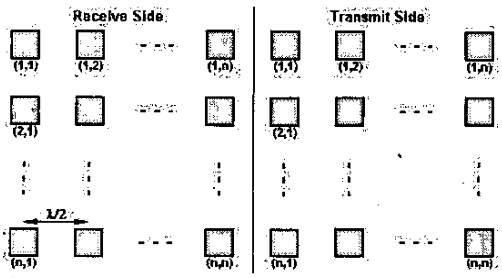

Fig 1. Arrangement of the antenna array elements

Fig. 2 shows the schematic of the noise-correlating radar with retrodirective antenna based on the heterodyne technique. The received RF signal is phase conjugated by mixing with the LO of frequency twice the RF signal. The IF signal is amplified by the power amplifier and re-transmitted. The signal received at element *(i, j)* is cross correlated with the signals at *(i+1, j)* and *(i, j+1)*. Correlation in the horizontal direction gives the *I* and *Q* components with the phase corresponding to azimuth angle ϕ. Similarly, correlation in the vertical direction gives the phase corresponding to elevation angle θ.

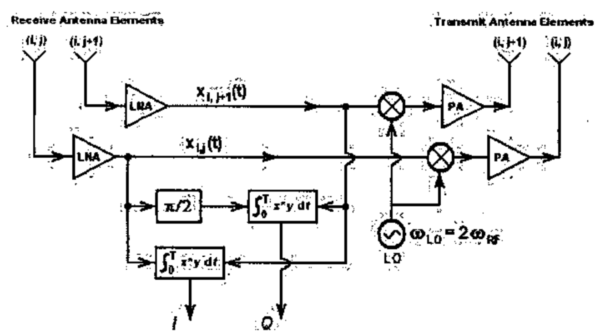

Fig 2. Schematic of the system for two adjacent elements

III. Analysis of The System

Let the instantaneous signal at the output of the LNA corresponding to antenna element *(i, j)* be $x_{ij}(t)$. Therefore $x_{ij}(t)$ contains some signal part $x_T(t)$ (which is any signal coming from the target, assuming only one target) plus some noise $n_{ij}(t)$, added at the receiver *(i, j)*. If the carrier frequency is ω_0, then we have

$$x_{ij}(t) = \mathrm{Re}\{[x_T(t) + n_{ij}(t)]\exp(-j\omega_0 t)\} \quad (1)$$

$$x_{i,j+1}(t) = \mathrm{Re}\{[x_T(t-\tau_\phi) + n_{i,j+1}(t)]\exp(-j\omega_0 (t-\tau_\phi))\} \quad (2a)$$

$$x_{i+1,j}(t) = \mathrm{Re}\{[x_T(t-\tau_\theta) + n_{i+1,j}(t)]\exp(-j\omega_0 (t-\tau_\theta))\} \quad (2b)$$

$$\tau_\phi = (\lambda/2)\cos(\phi)/c \quad (3a)$$

$$\tau_\theta = (\lambda/2)\cos(\theta)/c \quad (3b)$$

Here $x_T(t)$ and $n_{ij}(t)$ (for all *i*, *j*=1,2,...*n*) are band-limited zero-mean Gaussian-distributed complex envelopes. All of these are statistically independent and have real and imaginary parts as in-phase and quadrature-phase components. All elements receive the same signal $x_T(t)$, though phase shifted by an amount that depends on the direction from which the signal is coming. The variables $x_{ij}(t)$, $x_T(t)$, and $n_{ij}(t)$ are assumed to have variances σ^2_x, σ^2_{signal} and σ^2_{noise}, respectively. If $x_T(t)$ is non-zero, $x_{ij}(t)$ and $x_{i,j+1}(t)$ are correlated with cross-correlation coefficient (ρ) for signal of time duration T defined by

$$\rho\cos\phi = \frac{E[\frac{1}{T}\int_0^T x_{ij}(t)x_{i,j+1}(t)dt]}{E[\frac{1}{T}\int_0^T x_{ij}^2(t)dt]^{1/2}E[\frac{1}{T}\int_0^T x_{i,j+1}^2(t)dt]^{1/2}} \quad \frac{E[I]}{\sigma_x^2} \quad (4)$$

Similarly,

$$\rho\sin\phi \quad \frac{E[Q]}{\sigma_x^2} \quad (5)$$

Here *I* and *Q* are the in-phase and quadrature-phase components of the correlator output as shown in Fig.2. Whatever analysis holds for cross-correlation components along the horizontal direction of the array corresponding to angle ϕ, also holds for cross-correlation components along the vertical direction corresponding to angle θ. We will henceforth consider only the former case. We can also write for Gaussian distributed components

$$\sigma_x^2 = \sigma_{noise}^2 + \sigma_{signal}^2 = \sigma_{noise}^2(1+SNR) \quad (6)$$

From equations (4) and (6), we can show that

$$\rho\cos\phi = \sigma_{signal}^2\cos\phi/\sigma_x^2 \quad (7)$$

$$\Rightarrow \quad \rho = SNR/(1+SNR) \quad (8)$$

N independent samples of *I* and *Q* are added to get I_N and Q_N. The integration time for one independent sample is given by T_s=1/2*B, where B is the bandwidth. If the pulse duration is T_d, the number of independent samples for each pulse at every correlator output is T_d/T_s. For each received pulse, the signal $x_{ij}(t)$ is cross correlated with $x_{i,j+1}(t)$ for all i = 1, 2, ... *n*, and for all j = 1, 2, ... *n*-1. Therefore, for each pulse, we get *n(n-1)* correlator outputs giving independent *I*, *Q* samples. Summation can also be carried out for a multiple number of pulses because the *I*,

Q phases, which depend upon the direction of the target, are practically the same for all the pulses. Therefore, we can get a very large value of N. To get the probability of detection of the target, and the probability of false alarm, we define a random variable

$$Z_N = [I_N^2 + Q_N^2]^{1/2} \tag{9}$$

Using [6], Eq.21, we get the probability density function

$$p(Z_N) = \frac{2^{N+3}}{\sigma_x^2(1-\rho^2)(N-1)!}\left[\frac{Z_N}{\sigma_x^2}\right]^N$$

$$\times K_{N-1}\left[\frac{4Z_N}{\sigma_x^2(1-\rho^2)}\right] I_0\left[\frac{4\rho Z_N}{\sigma_x^2(1-\rho^2)}\right], \quad Z_N > 0 \tag{10}$$

Here K_N is the modified Bessel function of second kind of order N and I_m is the modified Bessel function of first kind of order m. The target is assumed to be detected when Z_N exceeds threshold T. The probability of detection of the target (P_d) and the probability of false alarm (P_f) for a particular threshold T can be evaluated as

$$P_d = p\,(Z_N > T), \qquad \rho \quad 0 \tag{11}$$

$$P_f = p\,(Z_N > T), \qquad \rho = 0 \tag{12}$$

Again, using [6], Eq.23, we get

$$P_d = p(Z_N > T) = \frac{2^{N+1}}{(N-1)!}\left[\frac{T}{\sigma_x^2}\right]^N \sum_{m=0} \rho^m$$

$$\times K_{N+m}\left[\frac{4T}{\sigma_x^2(1-\rho^2)}\right] I_m\left[\frac{4\rho T}{\sigma_x^2(1-\rho^2)}\right] \tag{13}$$

For $\rho = 0$, the above expression simplifies to

$$P_f = \frac{2^{N+1}}{(N-1)!}\left[\frac{T}{\sigma_x^2}\right]^N K_N\left[\frac{4T}{\sigma_x^2}\right] \tag{14}$$

Using equations (13) and (14), we can obtain the receiver operating (ROC) characteristics for the radar system, as shown in [6, Fig. 5]. The ROC curves are generated using Matlab, which fails to evaluate Bessel's function for very high values of N (as required in the proposed radar). We observe the fact that for lower values of ρ, increasing ρ by a factor k has almost the same effect as increasing N by a factor of k^2 and keeping ρ constant. Using this fact, we can generate the approximated ROC curves for higher values of N as shown in Fig. 3.

Now let us see how the SNR rises for the system after a few pulses. Let the link gain from the antenna to the target and back to the antenna element (without retrodirectivity) be G_L. Let the noise figure of each element receiver be NF, the noise power at the input of each LNA be N_0, and the power gain of the LNA be G_A (assuming G_A to be arbitrarily high). If the number of the transmit elements and receive elements is n_e (i.e. total number of elements is $2n_e$), the antenna gain due to retrodirectivity is n_e (array factor). For the transmission of the ith pulse, the signal variance (σ^2_{signal}) at the output of the LNA (Fig. 2) is assumed to be S_i and the noise variance (σ^2_{noise}) is N_i. The power transmitted by the antenna in each pulse is P_t, which can be controlled by the gain of the power amplifiers. Therefore the signal part of the transmitted power will experience a gain of $G_LG_An_e$, and the noise part a gain of G_LG_A (because noise is transmitted omni-directionally). Therefore, the signal and noise variances at the input of the power amplifier for transmission of the $(i+1)$th pulse are given by

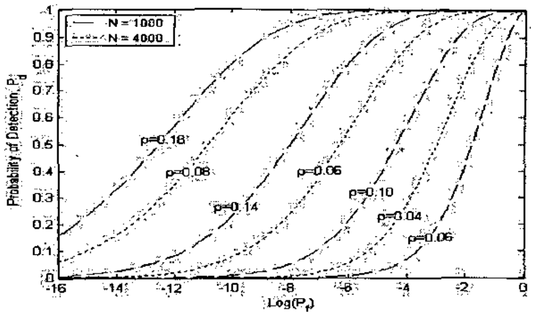

Fig 3. ROC curves for high values of N.

$$S_{i+1} = \frac{S_i}{S_i + N_i} \times G_LG_An_eP_t + \frac{N_i}{S_i + N_i} \times G_LG_AP_t \tag{15}$$

$$N_{i+1} = N_0 \times NF \times G_A = N_i, \qquad i > 0 \tag{16}$$

$$\Rightarrow SNR_{i+1} = \frac{S_{i+1}}{N_{i+1}} = \frac{SNR_i.n_e + 1}{SNR_i + 1} \times \frac{P_tG_L}{NF.N_0} \tag{17}$$

It should be noted that N_0 is the noise power at the input of each LNA, which is different from N_i. Since there is no signal coming from the target for the first transmitted pulse, we have $S_1 = 0$. That is, for the first pulse, only the noise present at the inputs of the LNAs is amplified and transmitted. Since it is uncorrelated for different elements, the power goes omni-directionally for the first pulse. The above equations and Eqn. 8 show that the SNR, and hence the correlation coefficient, rise to some asymptotic values after a few pulses, as shown in Fig. 4. If we have a pulsed phased array antenna with same number of elements ($2n_e$), the received SNR is

$$SNR_{phased\text{-}array} = P_t\,G_L\,(2n_e)^2 / NF.N_0 \tag{18}$$

One factor of $2n_e$ (array factor) corresponds to directivity gain. Another factor comes because received signals from all $2n_e$ elements are phased shifted and coherently added to get the required signal. In Fig. 4 and in the next section, all power numbers are compared with P_0, which is the transmitted power required for getting an SNR of 10 dB for the phased array radar.

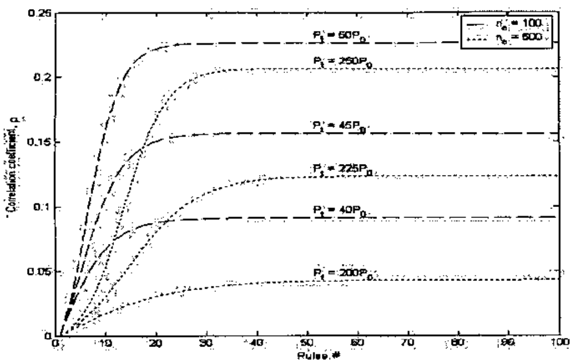

Fig 4. Correlation Coefficient between signals at LNA outputs.

IV. Comparisons with Phased Array Radar

Now we compare the performance of the retrodirective radar against a phased array radar with the same specifications in terms of size, total number of elements, element pattern etc. If we require a P_d of 0.9 and a P_f of 10^{-6}, using results from [7], we find that the SNR required for single pulse detection by a phased array radar is about 13 dB. This corresponds to a transmit power of $2P_0$. The acquisition time of a phased array radar will be the number of pencil beams required for complete scanning multiplied by the round trip time (T_r) of the pulses. For scanning the complete region defined by the element pattern gain, the number of pencil beams required is $2n_e$ (i.e. gain due to array factor). Therefore, for n_e=100, the acquisition time is roughly $200T_r$. For the same n_e, suppose the retrodirective radar transmits pulses at $50P_0$, each pulse being used for only one sample per correlator. This gives 10*9=90 samples per pulse, and we require about 11 pulses for N=1000. From Fig. 3 and 4, we can say that if the samples are integrated from the 7th to the 17th pulse, we can easily meet the above requirements of P_d and P_f. Therefore, the acquisition time is $17T_r$. This means that for an increase in power by factor of 25, we can reduce the acquisition time by a factor of about 12 using retrodirective radar. Similarly for n_e=500 we find that by increasing the power by a factor of 125, we can reduce the acquisition time by a factor of about 58 using the noise-correlation radar (assuming integration from pulses 9 to 17). We can do even better if the pulse widths are such that we get more than one sample per pulse from each correlator.

The direction of the target can be estimated by azimuth angle ϕ=arctan[Q_N/I_N] for correlations along horizontal direction of the array. Similarly we can get the elevation angle θ for correlations along the vertical direction. No special algorithms for tracking of the target are required as the retrodirectivity is "self-tracking". Our technique is also immune to any fluctuations in the target position etc. because detection depends only upon the power of the incoming signal and nothing else. But a disadvantage is that Doppler detection can't work. Our technique, with some additional algorithms, can also be used to detect and track multiple targets, as the retrodirective antenna transmits signal along all the directions from where the signal is coming.

V. Conclusions

A new retrodirective antenna based noise correlation search radar has been described in the paper. The radar system is shown to have a very low acquisition time as compared to a phased array radar with the same specifications except power. The acquisition time appears to scale almost inversely with the increase in power in the proposed radar when compared to a phased array system.

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